Assignment 7

Available Since: 12 June 2014  Due Date: 19 June 2014, 12:00 a.m.
You are permitted and encouraged to work in groups of two.

Exercise 1: General and s-t-flow networks 5 Points

Let \( N = (D = (V, A); l; u; b) \) be a general flow network with \( l(a) := 0 \) for all \( a \in A \).
Let \( N_{s,t} = (\bar{D} = (\bar{V}_{s,t}, \bar{A}_{s,t}); s; t; c) \) be the s-t-flow network with

\[
\begin{align*}
\bar{V}_{s,t} & = V \cup s, t \\
\bar{A}_{s,t} & = A \cup \{ (s, v) | v \in V, b(v) > 0 \} \cup \{ (v, t) | v \in V, b(v) < 0 \} \\
c(a) & = \begin{cases} u(a) & \forall a \in A \\
b(v) & \forall (s, v) \in \bar{A}_{s,t} \\
-b(v) & \forall (v, t) \in \bar{A}_{s,t} 
\end{cases}
\end{align*}
\]

(a) Sketch a general flow network \( N \) of reasonable size and complexity and also sketch the corresponding s-t-flow network \( N_{s,t} \).

(b) Show that a valid flow \( x \) in \( N \) exactly corresponds to a maximum s-t-flow \( x_{s,t} \) in \( N_{s,t} \) with value

\[
w(x_{s,t}) = \sum_{(s,v) \in \bar{A}_{s,t}} c((s,v)),
\]

and vice versa.

Exercise 2: Planar Straight-line Grid Drawing 15 Points

Implement a class lastname1.lastname2.a06.ShiftMethod to obtain a planar straight-line grid drawing for a triangulated plane graph.

(a) Implement a function to create a random triangulated planar graph for a given number of vertices \( n > 3 \) (use reasonably large \( n \)). Start with triangle \( v_1, v_2, v_n \) and successively add a vertex into a random inner face.

Use \( \text{createEdge(Node v, Edge e1, Node w, Edge e2, int d1, int d2)} \) or \( \text{changeEdge(Edge e, Edge e1, Edge e2, int d1, int d2)} \) from yfiles’ Graph class to specify the combinatorial embedding. Furthermore, use, e.g., \( \text{sortNodes(Comparator comp)} \) or \( \text{moveToFirst(Node v)} \), so that the generated graph returns the vertices in order \( v_1, v_2, v_n, \ldots \), i.e., the outer face of the graph appears at the start of the internal node list.
(b) Implement a function to determine a canonical ordering for such a graph. Label the nodes of the input graph with their ordering number, and return a corresponding array of nodes. To simplify the next part, sort the node order (as above) according to the canonical ordering \((v_1, v_2, v_3, \ldots v_n)\).

(c) Implement the shift method drawing algorithm to obtain a straight-line planar grid layout for the generated graph.