Assignment 8

Available Since: 19 June 2014   Due Date: 26 June 2014, 12:00 a.m.
You are permitted and encouraged to work in groups of two.

Exercise 1: Angles in drawings 10 Points

Prove the following statements

(a) The sum of inner angles of any triangle is $\pi$.

(b) The sum of inner angles of any simple convex $n$-gon is $(n - 2) \cdot \pi$.

(c) The sum of outer angles of any simple convex $n$-gon is $(n + 2) \cdot \pi$.

(d) The smallest angle of a triangulated regular $n$-gon is in $\Omega\left(\frac{1}{n}\right)$.

(e) The smallest angle between two distinct lines on the $n \times n$ integer grid is in $\Omega\left(\frac{1}{n^2}\right)$.
   
   Hint: Start with a line from $(0,0)$ to $(1,1)$. Find a second line creating a small angle.
   
   Argue to apply the approximation of trigonometric functions for small angles.

Exercise 2: Angular flow 4 Points

For the following graph $G$, construct and sketch the flow network $N_{s,t}(G)$, and compute and sketch a maximum flow as was shown in the lecture.
Exercise 3: Equivalence of Embeddings 6 Points

A graph is 3-connected if and only if between any two vertices there are at least 3 internally disjoint paths. Prove that a 3-connected simple planar graph $G = (V, E)$ has a unique planar embedding:

Consider a planar embedding of $G$ with face set $F$. Let $f \in F$ and let $C$ be the boundary of $f$, i.e., the sequence of edges and vertices around $f$. Let $G - C$ be the graph that is obtained from $G$ by deleting all vertices of $C$ and their incident edges. Show that

(a) $C$ is a simple cycle.

(b) $G - C$ is connected.

(c) $C$ is the boundary of exactly one face in any embedding of $G$. 