Assignment 11

Available Since: 10 July 2014  Due Date: 17 July 2014, 12:00 a.m.
You are permitted and encouraged to work in groups of two.

Exercise 1: Barycenter Heuristic 4 Points

Listed below is the barycenter heuristic for crossing reduction in a two-layer graph \( G = (L_1 \cup L_2, E) \), given fixed ordering \( x_1 \) (without final sorting of \( L_2 \)):

**Algorithm 1: Barycenter heuristic for a two-layer graph \( G = (L_1 \cup L_2, E) \)**

\[
\text{for } v = 1, \ldots, |L_2| \text{ do} \\
\quad b(v) \leftarrow \frac{1}{\deg_G(v)} \sum_{u \in N(v)} x_1(u); \\
\quad \text{if } b(v) = b(w), \ v \neq w \text{ then} \\
\quad \quad \text{separate } b(v) \text{ and } b(w) \text{ by a small amount;}
\]

Let \( bc(G, x_1) \) denote the number of crossings after applying the barycenter heuristic. Show that

\[
\text{opt}(G, x_1) = 0 \implies bc(G, x_1) = 0
\]

that is, the barycenter heuristic yields a crossing-free two-layer drawing of \( G \) if there exists a crossing-free two-layer drawing of \( G \) with fixed vertex order \( x_1 \).

Exercise 2: Sugiyama Framework 16 Points

Implement a class `lastname1_lastname2.a11.Sugiyama` to obtain a layered drawing for a given directed graph following steps 1. to 3. of the Sugiyama framework. Provide reasonably efficient implementations of the individual steps in separate classes:

(a) Implement improved greedy cycle removal in a class `CycleRemoval` to obtain a directed acyclic graph. Highlight reversed edges with color.

(b) Implement longest-path layering in a class `Layering`. Set y-coordinates of vertices corresponding to the layer assignment.

(c) Implement layer-by-layer sweep using the greedy switch heuristic (without restarting with different permutations for the lowest layer) in a class `CrossingReduction`. Remember to replace long edges by inserting dummy vertices along those edges beforehand. After crossing reduction, replace dummy vertices with bends (e.g., using `EdgeRealizer.appendBend(x,y)`).
Hints:

- Use `y.util.GraphHider` to temporarily hide graph elements instead of actually removing them.
- You may store information about reversed edges, layer assignment, dummy vertices etc. centrally, e.g., in an additional class, to make this information available for the individual classes above.
- Pseudocode for involved algorithms can be found on the lecture website.

Exercise 3: Horizontal Coordinate Assignment  

Hint: This is an optional exercise. It provides the opportunity to receive bonus points that increase your personal points total, but do not increase the amount of points required for qualification for the exam.

(a) Develop a heuristic for step 4. of the Sugiyama framework, i.e., horizontal coordinate assignment. Criteria for this step are:

- Final x-coordinates must respect the ordering from the crossing reduction step.
- Any vertex should preferably be balanced between its neighbors.
- Each long edge should have at most two bends, and all line segments of the edge except the first and last segment (inner segments) should be vertically aligned.

Argue how your heuristic meets those criteria.

(b) Implement your heuristic in a class `Coordinates` within the framework from exercise 2. Note that replacement of dummy vertices with bends should then only be performed after this step.