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Assignment 1

Post Date: 18 April 2012 **Due Date:** 02 May 2012, **8:15 am**
You are permitted and encouraged to work in groups of two.

Problem 1: Party — 2 Points

Show that at a party the number of guests that shakes hands with an odd number of other guests is even.

Problem 2: Trees — 10 Points

A *tree* is a connected graph that does not contain any cycle.

(a) Show that a tree with $|V| \geq 2$ vertices has at least two *leaves*, i. e., vertices of degree 1.

(b) Let $T = (V, E)$ be a tree with $|V| \geq 2$ vertices and let $u \in V$ be a leaf. Show that $T' = (V \setminus \{u\}, E)$ is a tree.

(c) Let $T = (V, E)$ be a tree. Show that $|E| = |V| - 1$.

(d) Let $G = (V, E)$ be a graph. Show the equivalence of the following properties:

   i. G is connected and acyclic.
   ii. There is a unique simple path between any two vertices in G.
   iii. G is connected and for each $e \in E$ the subgraph $G' = (V, E \setminus \{e\})$ is disconnected.
   iv. G is acyclic and for each $e \in \binom{V}{2} \setminus E$ the graph $G' = (V, E \cup \{e\})$ contains a cycle.

Problem 3: Depth-First Search — 4 Points

Starting at a vertex s, *depth-first search* traverses vertices and edges as far as possible before backtracking and returning to the most recent vertex that has still unexplored neighbors. Algorithm 1 shows a detailed description of depth-first search.

[please turn over]

Algorithm 1: Depth-First Search

Input: connected graph $G = (V, E)$, vertex $s \in V$
Data: stack S, counter d
Output: depth-first numbers `DFS` (at the beginning ∞)
predecessor `parent` (at the beginning `nil`)

```
begin
    DFS[s] ← 1;
    d ← 2;
    mark s;
    push s → S;
    while S ≠ ∅ do
        v ← top(S);
        if there is an unmarked edge {v, w} ∈ E then
            mark {v, w};
            if w unmarked then
                DFS[w] ← d;   d ← d + 1;
                mark w;   push w → S;
                parent[w] ← v;
        else pop v ← S;
end
```

In Algorithm 1, all vertices $v \in V$ receive a finite depth-first search number `DFS`$[v]$. If in the end `parent`$[w] = v$, then $\{v, w\}$ is called *tree edge*, all other marked edges are called *back edges*.

Prove that the graph induced by the set of tree edges is a *spanning tree* of the connected graph G, i. e., a tree that contains all vertices of G.

Problem 4: Search Strategies **4 Points**

Start at the black vertex and perform

(a) a breadth-first search and

(b) a depth-first search.

Number the vertices and edges in the order in which they are visited.