University of Konstanz
Department of Computer & Information Science
PD Dr. Sabine Cornelsen / Melanie Badent

Algorithmic Graph Theory
SS 2012

Assignment 2

Post Date: 09 May 2012 **Due Date:** 22 May 2012
You are permitted and encouraged to work in groups of two.

Problem 1: Dating Agency I — 8 Points

Imagine you are a matchmaker with n male and n female clients. Each man has given a ranking for the n women, and vice versa. A higher ranking implies a greater preference. Your job is to arrange n datings such that there is no undated pair that perfers each other to their current date.

Develop an algorithm that solves the dating problem, i. e., it constructs a matching with the given properties. Prove that your algorithm works correctly and analyze its running time.

Test your algorithm with the example given below.

	W1	W2	W3	W4	W5
M1	3	5	2	1	4
M2	4	3	5	1	2
M3	4	1	3	2	5
M4	1	3	2	5	4
M5	4	2	3	1	5

	M1	M2	M3	M4	M5
W1	5	4	3	1	2
W2	5	1	3	2	4
W3	5	4	1	3	2
W4	5	3	1	2	4
W5	5	3	2	1	4

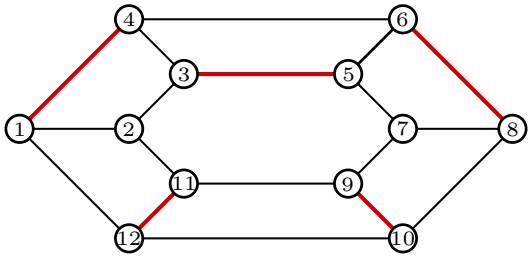
Problem 2: Dating Agency II — 3 Points

Again you are a matchmaker with n male and n female clients. Now, each man has given a list of at most n women that he wants to date. Transform this problem into a maximum flow problem such that you can arrange as many dates as possible.

[please turn over]

Problem 3: Alternating and Augmenting Paths **5 Points**

Consider the following graph together with the given matching indicated by the bold, red edges.



Find in this graph

(a) an alternating path of length 10,

(b) an alternating cycle of length 10,

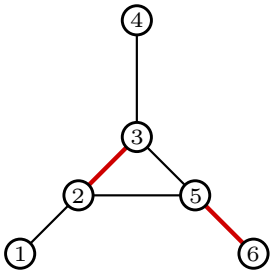
(c) an augmenting path of lenght 5,

(d) an augmenting path of lenght 9, and

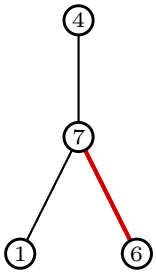
(e) an alternating spanning tree rooted at vertex 2.

Problem 4: Blossoms **4 Points**

Consider the graph given in Figure (a) that has an augmenting path $\langle 1, 2, 3, 4\rangle$. The sequence $\langle 2, 3, 5, 2\rangle$ is a blossom in the graph. Contracting these vertices yields the graph in Figure (b). However, this graph does not have an augmenting path starting at vertex 1.



(a)



(b)

Is this a counterexample for the theorem that a graph has an augmenting path from a vertex v if and only if the contracted graph has an augmenting path from v?