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Department of Computer & Information Science
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Algorithmic Graph Theory
SS 2012

Assignment 3

Post Date: 23 May 2012 **Due Date:** 29 May 2012
You are permitted and encouraged to work in groups of two.

Problem 1: Flow Constraints — 3 Points

Let f be a flow in a network and let $\alpha \in \mathbb{R}$. The product αf is defined as $(\alpha f)(u, v) = \alpha \cdot f(u, v)$. Show that the flows in a network form a *convex set*, i.e., if f_1 and f_2 are flows, then $\alpha f_1 + (1 - \alpha) f_2$, $0 \leq \alpha \leq 1$, is a flow.

Problem 2: Minimum s-t-Cut — 3 Points

(a) Find an efficient method how to compute a minimum s-t-cut of a flow network if a maximum flow is given.

(b) Find an algorithm that computes a minimum s-t-cut of a flow network where the cut should have a minimum number of arcs.

Advice: Modify the capacities suitably.

Problem 3: Multi-Source-Multi-Sink Network — 4 Points

Let $D = (V, A)$ be a simple directed graph with arc capacities $u\colon A \to \mathbb{R}_{\geq 0}$ and let $S, T \subset V$ be two subsets of V with $S \cap T = \emptyset$. We refer to the tuple $N = (D, u, S, T)$ as *multi-source-multi-sink network*.

A mapping $f : A \to \mathbb{R}_{\geq 0}$ is a *flow* in N if it fulfills the arc capacity constraint for all arcs $(v, w) \in A$ and the flow conservation constraint for all vertices $v \in V \setminus (S \cup T)$. The *value* $w(f)$ of a flow f in N is defined as $w(f) := \sum_{s \in S} \left(\sum_{(s,v) \in A} f(s, v) - \sum_{(v,s) \in A} f(v, s) \right)$.

Describe how the problem of determining a maximum flow in N can be reduced to the problem of determining a maximum flow in a single-source-single-sink network. Show that this method indeed computes a maximal flow in N.