University of Konstanz
Department of Computer & Information Science
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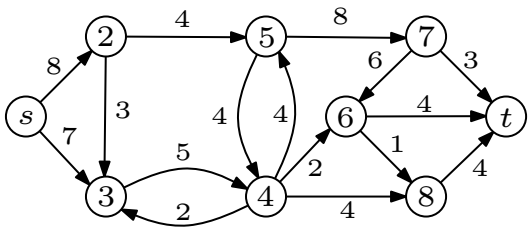
Algorithmic Graph Theory
SS 2012

Assignment 4

Post Date: 30 May 2012 **Due Date:** 12 June 2012
You are permitted and encouraged to work in groups of two.

Problem 1: Ford & Fulkerson — 4 Points

Let (D, u, s, t) be the following flow network where the numbers at the edges indicate their capacity:



Find a maximum s-t-flow and a minimum s-t-cut using the algorithm of Ford & Fulkerson. Indicate in each step the augmenting path and the value of the augmentation.

Problem 2: Entrepreneur — 6 Points

Imagine you are an entrepreneur with the following problem: In each of T periods, you can buy, hold for later sale, or sell some of your goods. In each period i you can buy at most a_i units of the commodity, hold at most b_i units over for the next period, and you must sell at least c_i units. Assume that d_i, e_i, and f_i denote the purchase cost, inventory carrying cost, and selling price per unit in period i. You want to create a buy-sell policy such that the total profit is maximized in the T periods. Formulate this problem as a minimum cost flow problem for $T = 4$. Assume that you start with no commodities and that you cannot sell the commodity in the same period in that you bought it.

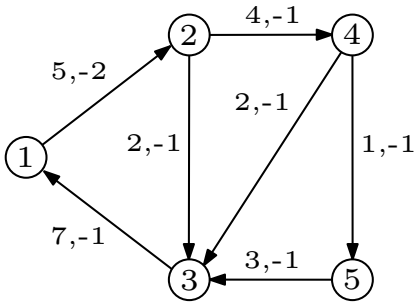
[please turn over]

Problem 3: Cycle Canceling I **6 Points**

Compute a flow of minimum cost of the following network by applying the cycle canceling algorithm. Use the zero flow as the starting solution.

Indicate optimal vertex potentials that prove that the flow has indeed minimum cost.

The first number at an edge indicates the upper bound on flow, the second number at an edge indicates the costs per unit of the flow. For all vertices, supply and demand equals zero, respectively.



Problem 4: Cycle Canceling II **4 Points**

Show that if the cycle canceling algorithm is applied to the network shown below, some sequence of augmentations requires 200 iterations.

The first number at an edge indicates the upper bound on flow, the second number at an edge indicates the costs per unit of the flow. For all vertices, supply and demand equals zero, respectively.