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Department of Computer & Information Science
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Algorithmic Graph Theory
SS 2012

Assignment 6

Post Date: 27 June 2012 **Due Date:** 10 July 2012
You are permitted and encouraged to work in groups of two.

Problem 1: Euler Tour — 3 Points

(a) Show that the symmetric difference $C_1 + C_2$ of two cycles C_1 and C_2 is again a cycle.

(b) An *Euler tour* in a connected graph G is a sequence $\langle e_1, \ldots, e_m \rangle$ of edges that traverses each edge of G exactly once and such that e_m and e_1 as well as any two consecutive edges e_i and e_{i+1}, $i = 1, \ldots, m-1$ are incident. Show that an Euler tour in G exists if and only if each vertex of G has even degree.

Problem 2: Cycle Bases — 9 Points

Let $G = (V, E)$ be an undirected, connected graph with m edges. Let $T = (V, E_T)$ be a spanning tree of G. For each non-tree edge $e = \{v, w\} \in E \setminus E_T$ we define a *fundamental cycle* $C_e = \{e\} \cup P_e$ where P_e is the set of edges on the unique path in T between v and w. Show that the *fundamental cycle basis* $\mathcal{B}_T = \{C_e \mid e \in E \setminus E_T\}$ with respect to T is indeed a cycle basis of the cycle space $\mathcal{C}$:

(a) Show that $\mathcal{B}_T$ is linearly independent.

(b) Show that $\mathcal{B}_T$ is a generating system of $\mathcal{C}$ by describing how an arbitrary cycle can be written as a linear combination of elements of $\mathcal{B}_T$.

Hint: To prove that your linear combination indeed creates the desired cycle, it might help to consider the cuts of G induced by the connected components of T minus one edge.

(c) Show that $|\mathcal{B}_T| = m - n + 1$, where $|V| = n$ and $|E| = m$.

Problem 3: Number of Cycles — 3 Points

Find an (infinite) family of graphs $(G_i)_{i \in I}$ with an increasing number edges such that the number of cycles in G_i grows exponentially in the number of edges. Explain each of your steps in detail.

[please turn over]

Problem 4: Algorithm of de Pina **5 Points**

Algorithm 1: Algorithm of de Pina

Input: connected graph $G = (V, E)$, edge weights $w : E \to \mathbb{R}_{\geq 0}$
Data: edge sets S_i, $i = 1, \ldots, \nu$
Output: minimum cycle basis $\mathcal{B} = \{C_1, \ldots, C_\nu\}$

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begin
    Compute spanning tree of G with non-tree edges e_1, ..., e_ν;
    for i ← 1, ..., ν do
        S_i ← {e_i};
    for i ← 1, ..., ν do
        find minimum weight cycle C_i with |C_i ∩ S_i| odd;
        for j ← i, ..., ν with |C_i ∩ S_j| odd do
            S_j ← S_j + S_i;
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Find a minimum cycle basis of the graph below (all edges have capacity 1) using the algorithm of de Pina. Use the tree indicated by the bold edges and process the edges in the given order. Document each of your steps in detail.