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Combinatorial Optimization
Summer 2014

Assignment 2

Post Date: 2 May 2014 **Due Date:** 9 May 2014 **Tutorial:** 14 May 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Convex Sets — 7 Points

In a *convex set* $S \subset \mathbb{R}^n$, the line between two elements $a, b \in S$ is also in S. Formally $\forall a, b \in S, \forall t \in [0,1] : (1-t)a + tb \in S$.

A *convex function* f is a function in which every point on a line between two points of the graph of the function lies above the graph: $\forall x_1, x_2 \in X, \forall t \in [0,1] : f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2)$.

A local minimum of a function $f : \mathbb{R}^n \to \mathbb{R}$ is a point $a \in \mathbb{R}^n$ such that all points b in a sufficiently small ϵ-neighborhood $N_\epsilon(a)$ of a are greater than a.

(a) Show that the set of feasible solutions of a linear program is convex.

(b) Show that a local minimum of a convex function is also a global minimum.

Problem 2: Pivot-Operations — 8 Points

Consider the following linear program in slack form:

$$
\begin{array}{lrcrcrcrcrcr}
\text{maximize} & 10x_1 & - & 57x_2 & - & 9x_3 & - & 24x_4 & & & = & f(x) \\
\text{subject to} & 0.5x_1 & - & 5.5x_2 & - & 2.5x_3 & + & 9x_4 & + & x_5 & = & 0 \\
& 0.5x_1 & - & 1.5x_2 & - & 0.5x_3 & + & x_4 & + & x_6 & = & 0 \\
& x_1 & & & & & & & + & x_7 & = & 1 \\
& & & & & x_1, x_2, x_3, x_4, x_5, x_6, x_7 & & & & & \geq & 0
\end{array}
$$

(a) Apply seven pivoting-steps of the Simplex Algorithm. Choose the variables for the pivoting-steps as follows:

- as *entering* non-basic variable x_e choose the variable with the greatest coefficient in the objective function
- as *leaving* variable x_l choose the basic variable with the smallest index that is a candidate for pivoting

Example: In the first step, swap the roles of x_1 (because 10 is the greatest coefficient) and x_5 (because 5 is the smallest index in whose row the value of x_1 has a positive coefficient)

(b) How many more steps would it take for the algorithm to terminate if the rules for choosing the pivot-variables remain unchanged?

Problem 3: Running Time **5 Points**

Let n be the number of variables and m be the number of constraints. What is the asymptotic running time of one iteration in the Simplex Algorithm (including the selection of the entering and leaving variables and the pivoting operation)?