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Combinatorial Optimization
Summer 2014

Assignment 3

Post Date: 9 May 2014 **Due Date:** 16 May 2014 **Tutorial:** 21 May 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Simplex — 8 Points

Solve the following linear program with the Simplex-Algorithm. (Note that you have to find a feasible solution with an auxiliary linear program first.)

$$\begin{array}{lrcrcr} \text{maximize} & x_1 & + & 3x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & 8 \\ & -x_1 & - & x_2 & \leq & -3 \\ & -x_1 & + & 4x_2 & \leq & 2 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

Problem 2: Single Variable Linear Program — 7 Points

For $a, b, c, x \in \mathbb{R}$ consider the following simple linear program P.

$$\text{maximize} \qquad cx$$

subject to

$$ax \leq b \qquad \text{and} \qquad x \geq 0.$$

Let further D be the dual of P.

(a) State for which values of a, b, and c, the linear programs P and D, respectively,

- are infeasible
- are unbounded
- have an optimal solution with finite objective value

Relate the cases.

(b) Prove or disprove that in general the dual of an unbounded linear program is infeasible.

Problem 3: Complementary Slackness **5 Points**

Let $\overline{x}$ be a feasible solution to the primal linear program

$$\text{maximize} \qquad \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, \ldots, m \qquad \text{and} \qquad x_j \geq 0, j = 1, \ldots, n$$

and let $\overline{y}$ be a feasible solution to the corresponding dual linear program. Prove that $\overline{x}$ and $\overline{y}$ are optimal if and only if

$$\sum_{i=1}^{m} a_{ij} \overline{y}_i = c_j \text{ or } \overline{x}_j = 0 \quad \text{ for } j = 1, \ldots, n$$

and

$$\sum_{j=1}^{n} a_{ij} \overline{x}_j = b_i \text{ or } \overline{y}_i = 0 \quad \text{ for } i = 1, \ldots, m.$$