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Combinatorial Optimization
Summer 2014

Assignment 4

Post Date: 16 May 2014 **Due Date:** 23 May 2014 **Tutorial:** 28 May 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Comparing Distributions — 7 Points

A method to compare two multidimensional distributions that is often used as a measure for the similarity of images works as follows:

Let P_1 and P_2 be two distributions (e.g. histograms in the case of image similarities) over a set of n bins. We define $w_{i,j}$ as the weight that bin j has in distribution i. Furthermore, we have a distance matrix D in which the distance of bins k and l from each other is given by $d_{k,l}$. In order to compare the distributions, we measure how much *work* is needed to transform one distribution into the other.

Figuratively, we can imagine n sites in space such that the distance between site k and l is $d_{k,l}$. At each site, there is a blue and a red bucket. The total volume of all blue buckets is equal to the total volume of the red buckets. The blue buckets are filled with sand and represent the bins in P_1, the red buckets are empty and represent the bins in P_2. At site number j, the blue bucket contains $w_{1,j}$ units of sand and the red bucket has room for exactly $w_{2,j}$ units of sand. The task is to completely fill the red buckets with sand from the blue ones. Since the buckets have different sizes, it might be necessary to use sand from different sites in order to fill the red bucket at a site. The work needed to transfer one unit of sand from site k to site l corresponds to the distance $d_{k,l}$.

Formulate a linear program that minimizes the work needed to fill the red buckets (i.e. the cost of transforming one distribution into the other).

Problem 2: Total Unimodularity — 5 Points

Let the matrix $A \in \mathbb{R}^{n \times m}$ be totally unimodular. Show that the matrix $\binom{A}{-A} \in \mathbb{R}^{2n \times m}$ is also totally unimodular.

Please turn over

Problem 3: Total Unimodularity II **8 Points**

The incidence matrix of an undirected graph $G = (V, E)$ is a matrix $B \in \mathbb{R}^{|V| \times |E|}$ that has a row for every vertex $v \in V$ and a column for every edge $e \in E$, such that

$$b_{ij} = \begin{cases} 1, & \text{if edge } j \text{ is incident to vertex } i \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that the incidence matrix of a bipartite graph is totally unimodular.

(b) Does this hold for the incidence matrix of every undirected graph?