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Combinatorial Optimization
Summer 2014

Assignment 8

Post Date: 13 June 2014 **Due Date:** 20 June 2014 **Tutorial:** 25 June 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Successive Shortest-Paths — 5 Points

Consider successive shortest path on the equivalent min-cost max-flow network. Always perform a single source shortest path in the residual network regarding the reduced costs from the global source s. Always augment on a path with minimum costs in the residual network from s to the global sink t.

Show that the following holds after each update of the potentials and before the following augmentation of the flow:

The vertex potentials always correspond to the shortest path distances from s in the residual network regarding the original costs.

Problem 2: Matchings on Bipartite Graphs — 7 Points

A *minimum vertex cover* of a graph $G = (V, E)$ is a set of nodes $C \subseteq V$ of minimal size, such that every edge $e \in E$ is incident to one of the nodes in C. Formally: $\forall (v, v') \in E : C \cap \{v, v'\} \neq \emptyset$.

(a) Show that a cardinality maximum matching on a bipartite graph has the same size as a minimum vertex cover. (Hint: Argument via the dual LP.)

(b) How about non-bipartite graphs?

Please turn over

Problem 3: Perfect Matchings **8 Points**

Let T be a tree and $G = ((A, B), E)$ be a bipartite graph.

(a) Show that T has at most one perfect matching.

(b) Show that if $|A| = |B| = n$ and $|E| > (k-1)n$ then there exists a matching M on G with $|M| \geq k$.

Let now M be a matching on G. We transform G in a directed network D by directing an edge from A to B if it does not belong to M and from B to A if it does belong to M.

(c) Show that there exists an augmenting path in G with respect to M if and only if there exists a directed path in D between an unmatched vertex in A and an unmatched vertex in B.