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Combinatorial Optimization
Summer 2014

Assignment 10

Post Date: 27 June 2014 **Due Date:** *3 July 2014* **Tutorial:** 9 July 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Unmatchable Edges — 7 Points

Let $G = (V, E)$ be a graph. We call an edge $e = \{u, v\} \in E$ *unmatchable*, if no perfect matching on G contains e. Let now G^{uv} be the graph G without u and v and all their adjacent edges.

(a) Show that e is unmatchable if and only if G^{uv} has no perfect matching.

(b) Give a polynomial time algorithm that finds all unmatchable edges.

(c) Give an $\mathcal{O}(|V|^2|E|)$-time algorithm that finds all unmatchable edges.

(d) BONUS: Give an $\mathcal{O}(|V|^3)$-time algorithm that finds all unmatchable edges.[1]

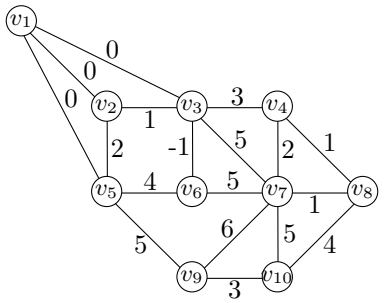
(Hint: Find a perfect matching first and then find all unmatchable edges for each node.)

Problem 2: — 5 Points

Give a method to find a feasible assignment to the dual variables of the minimum perfect matching problem. (Note that edge weights can be both positive and negative.)

Problem 3: — 8 Points

Find a minimum-weight perfect matching for the graph below using the algorithm from the lecture notes.



[1] Of course a solution for (c) implies a solution for (b), and a solution for (d) implies a solution for (c)...