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Assignment 11

Post Date: 4 July 2014 **Due Date:** 11 July 2014 **Tutorial:** 18 July 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Independent Vertex Sets — 4 Points

Let $G = (V, E)$ be a graph. Let $\mathcal{I} = \{V' \subseteq V;\ \{u, v\} \notin E \text{ for } u, v \in V'\}$.

(a) Show that $(V, \mathcal{I})$ is an independence system.

(b) Is $(V, \mathcal{I})$ also a matroid?

Problem 2: Matching Matroid — 6 Points

Let $G = (V, E)$ be a graph. Let

$$\mathcal{I} = \{V' \subseteq V;\ \text{there is a matching } M \text{ of } G \text{ s.t. no vertex in } V' \text{ is free}\}.$$

Show that $(V, \mathcal{I})$ is a matroid.

Problem 3: Weight Sequence — 4 Points

Let $(X, \mathcal{I})$ be a matroid and let $\omega : X \longrightarrow \mathbb{R}$ be a weight function. The *weight sequence* of a basis $B = \{x_1, \ldots, x_d\}$ of $(X, \mathcal{I})$ is the sequence $\langle \omega(x_1), \ldots, \omega(x_k) \rangle$ of weights of the elements of B ordered such that $\omega(x_1) \leq \cdots \leq \omega(x_k)$.

Show that any two minimum weight bases of a matroid have the same weight sequence.

[please turn over]

Problem 4: Fundamental Cycle Basis **6 Points**

Let $G = (V, E)$ be an undirected, connected graph with m edges. Let $T = (V, E_T)$ be a spanning tree of G. For each non-tree edge $e = \{v, w\} \in E \setminus E_T$ we define a *fundamental cycle* $C_e = \{e\} \cup P_e$ where P_e is the set of edges on the unique path in T between v and w. Show that the *fundamental cycle basis* $\mathcal{B}_T = \{C_e \mid e \in E \setminus E_T\}$ with respect to T is indeed a cycle basis of the cycle space $\mathcal{C}$:

(a) Show that $\mathcal{B}_T$ is linearly independent.

(b) Show that $\mathcal{B}_T$ is a generating system of $\mathcal{C}$ by describing how an arbitrary simple cycle can be written as a linear combination of elements of $\mathcal{B}_T$.

Hint: Consider the cuts of G induced by the connected components of T minus one edge. Observe that a cycle crosses a cut an even number of times.