University of Konstanz
Department of Computer & Information Science
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Combinatorial Optimization
Summer 2014

Last Assignment

Post Date: 11 July 2014 **Due Date:** — **Tutorial:** 23 July 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Linear Programming and the Greedy Algorithm

Let $(X, \mathcal{I})$ be a matroid with $X = \{1, \ldots, n\}$. For $A \subseteq X$ let $r(A) = \max\{|I|;\ I \in \mathcal{I}, I \subseteq A\}$. Let $\omega \in \mathbb{R}^n$. Assume that $\omega_i \leq \omega_j$ for $i < j$. Consider the following linear programming formulation of the minimum weight basis problem. Find an indicator vector $x \in \mathbb{R}^n$

$$\text{minimizing} \quad \omega^T x$$

subject to

$$\begin{array}{rcll} \sum_{i \in X} x_i & = & r(X) & \\ \sum_{i \in A} x_i & \leq & r(A), & A \subsetneq X \\ x_i & \geq & 0, & i \in X \end{array}$$

(a) Show that the indicator vector of a basis of $(X, \mathcal{I})$ is a feasible solution.

(b) Formulate the dual linear program.

(c) Show that setting

$$\pi(A) = \begin{cases} \omega_{i+1} - \omega_i & \text{if } A = \{1, \ldots, i\}, i = 1, \ldots, n-1 \\ -\omega_n & \text{if } A = X \\ 0 & \text{otherwise} \end{cases}$$

for $A \subseteq X$ yields feasible dual variables.

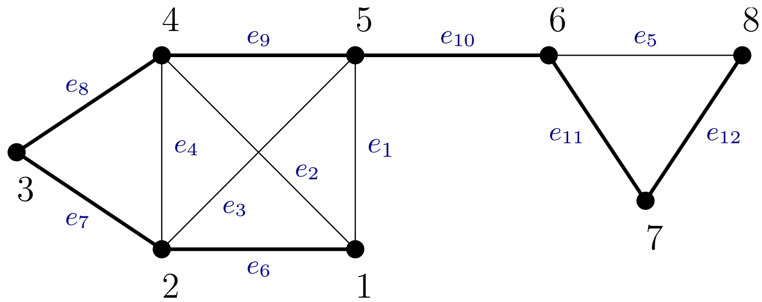
(d) Formulate the complementary slackness conditions.

(e) Let B be the basis constructed by the greedy algorithm. Show that the variables defined in (c) fulfill the complementary slackness conditions.

[please turn over]

Problem 2: Algorithm of de Pina

Find a minimum cycle basis of the graph below (all edges have weight 1) using the algorithm of de Pina. Use the tree indicated by the bold edges and process the edges in the given order.



Problem 3:

Formulate questions concerning the lecture.