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Algorithms for Planar Graphs
Summer 2017

Assignment 1

Post Date: 24 April 2017 **Due Date:** 08 May 2017 **Tutorial:** 10 May 2017

Problem 1: Different Embeddings — 3 Points

Let G be a planar graph with a planar embedding with f faces. The *degree* of a face f_i is the number a_i of edges that are incident to f_i (counting bridges twice). Assume that the faces $f_1, \ldots, f_f$ are ordered such that their degrees are non descending. Consider the face-degree-sequence $(a_1, \ldots, a_f)$.

Can a planar graph have two embeddings with different face-degree-sequences?

Problem 2: Euler's Formula — 4 Points

Let n be the number of vertices, m the number of edges, f the number of faces and k the number of connected components of a planarly embedded simple graph G.

(a) Prove by induction on m that $f - m + n - k = 1$.

(b) Conclude that $m \leq 2n - 4$ and $f \leq n - 2$ if G is bipartite and $n \geq 3$.

Problem 3: Vertex Coloring — 3 Points

Give a linear time algorithm that colors the vertices of a planar graph with at most six colors such that no two adjacent vertices have the same color.