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Algorithms for Planar Graphs
Summer 2017

Assignment 8

Post Date: 19 June 2017 **Due Date:** 26 June 2017 **Tutorial:** 05 July 2017

Problem 1: MacLane's Planarity Criterion — 6 Points

Let G be a biconnected plane graph and let $\mathcal{B}_I$ be the set of the inner facial cycles of G.

(a) Show that $\mathcal{B}_I$ generates the cycle space of G.

(b) Show that $|\mathcal{B}_I|$ equals the dimension of the cycle space.

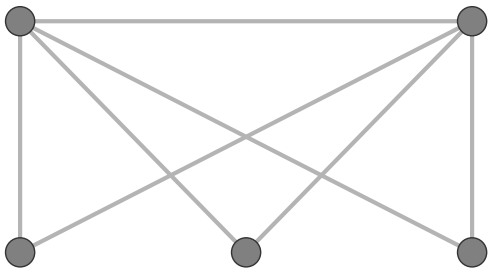
(c) Show that K_5 does not have a 2-basis.

(d) Show that $K_{3,3}$ does not have a 2-basis.

Hint for (c) *and* (d): Add a linear combination of the elements to the cycle basis and use a counting argument.

Problem 2: Cycle and Cut Bases — 4 Points

Consider the following graph $G = (V, E)$:



(a) Find a basis for the cycle space, which is not a fundamental cycle basis with respect to any spanning tree of G.

(b) Is there a subset of E that is both, a cycle and a cut?

(c) Express E as the symmetric difference of a cycle and a cut. Recall that cuts must be non-empty.

(d) Give a basis of the cut space of G.