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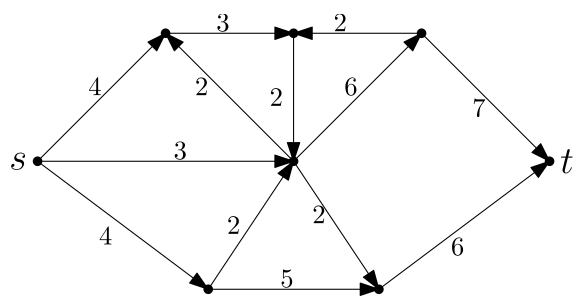
Algorithms for Planar Graphs
Summer 2017

Assignment 9

Post Date: 26 June 2017 **Due Date:** 03 July 2017 **Tutorial:** 05 July 2017

Problem 1: Max Flow on s-t-Planar Graphs — 4 Points

Solve the maximum flow problem in the following s-t-planar network by applying



(a) Hassin's algorithm, and

(b) the uppermost path algorithm.

[please turn over]

Problem 2: Uppermost Path Algorithm **6 Points**

Consider a planar drawing of an s-t-planar network $(D = (V, E), s, t, c)$ such that s is the rightmost point in the drawing and t is the leftmost point in the drawing. Now consider the following algorithm.

Algorithm 1: Uppermost Path

```
Initialize f ← 0;
while there is a directed s-t-path do
    Let P be the edge set of the uppermost directed s-t-path;
    Δ ← min_{e∈P}(c(e) − f(e));
    for e ∈ P do
        f(e) ← f(e) + Δ;
        if f(e) = c(e) then
            remove e from the network;
```

(a) Show that the uppermost path algorithm computes a maximum flow on an s-t-planar network.

(b) Does the corresponding algorithm also guarantee a maximum flow if you choose any (not necessarily uppermost) directed s-t path P.

(c) Does the flow computed by the uppermost path algorithm equal the flow computed by the algorithm of Hassin?

Hint: You can use without proof that the capacity of a minimum s-t-cut equals the value of a maximum s-t-flow.