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Algorithms for Planar Graphs
Summer 2017

Assignment 10

Post Date: 03 July 2017 **Due Date:** 10 July 2017 **Tutorial:** 19 July 2017

Problem 1: Venkatesan's Approach — 6 Points

Let $(D = (V, E), s, t, c)$ be a planar bidirected flow network. Choose a directed s-t-path P of D. For $(v, w) \in V \times V$, set $\pi(v, w) = 1$ if $(v, w) \in P$, $\pi(v, w) = -1$ if $(w, v) \in P$, and $\pi(v, w) = 0$ otherwise. Let $\lambda \in \mathbb{R}_0^+$ be such that the directed dual graph D^* with edge length $\ell_\lambda(e^*) = c(e) - \lambda\pi(e)$, $e^* \in E^*$ does not contain a negative directed cycle, i.e., such that the shortest path distances $d_\lambda(v^*, w^*)$ in D^* with respect to ℓ_λ are well defined. Choose an arbitrary vertex s^* of D^*. Prove that

$$\phi_\lambda(e) = max(0, d_\lambda(s^*, \text{right}(e)) - d_\lambda(s^*, \text{left}(e)) + \lambda\pi(e))$$

is a flow in D with value λ, i.e. show that the following properties are fulfilled:

(a) capacity constraint

(b) flow conservation, and

(c) $w(\phi_\lambda(e)) = \lambda$.

Problem 2: Separators of Trees — 4 Points

Let T be a tree with non-negative weights on the vertices that sum to one. A *weighted vertex separator* of T is a partition of the vertex set into two sets A and B of weight at most $2/3$ and a vertex v such that there is no edge between A and B.

(a) Show how to compute a weighted vertex separator of a tree in linear time.

(b) Can the vertex set of any tree with non-negative weights on the vertices summing to one be partitioned into two sets A and B of weight at most $1/2$ and a vertex v such that there is no edge between A and B?