UNIVERSITY OF KONSTANZ ALGORITHMICS GROUP V. Amati / J. Lerner / B. Nick Network Modeling Winter Term 2011/2012

# Assignments $\mathcal{N}^{\underline{\circ}}$ 1 - part i

released: 26.10.2011 due: 02.11.2011, 14:15h (solutions can be handed over at the beginning of the lecture)

#### Task 1: Structural Balance

# 5 points

Let  $\Delta$  the set of undirected triangle graphs, in which each edge is either labeled positive or labeled negative — that is, instead of being present or not, each of the three edges is either positive or negative. Define a random graph model on  $\Delta$ , such that all of the following conditions hold:

- (1) all balanced graphs are more probable than unbalanced ones
- (2) all edges are pairwise independent
- (3) every edge depends on the two others

# Task 2: Valid Probability Spaces2 points

Use the binomial theorem to show that  $\mathcal{G}(n,p)$  is indeed a valid random graph model, i.e.  $\sum_{G \in \mathcal{G}} P(G) = 1$ .

# Task 3: Expected Number of Motifs3 points

We saw that the expected number of edges in  $\mathcal{G}(n,p)$  models is  $\mathbb{E}[m] = p\binom{n}{2}$ .

What is the expected number of triangles, and, more general, the expected number of induced k-circles? (i.e. k nodes whose incident edges form a k-circle without shortcuts)