

Assignments $\mathcal{N}^o$ 2 - PART II

released: 16.11.2011 **due:** 22.11.2011, 10AM

Task 1: Inappropriate Sampling from ERGMs

4 points

So far, algorithms for sampling from $G(n, p)$ or the preferential attachment model decided about the inclusion/exclusion of edges one after the other.

Provide a (preferably simple) example which demonstrates that a corresponding strategy does not work out for the ERGM class in general, i. e. starting with the empty edge set $E = \emptyset$ and sequentially adding edge e to E with probability $\frac{P(V, E \cup \{e\})}{P(V, E) + P(V, E \cup \{e\})}$ yields incorrect outcomes — you are allowed to reuse probability calculations from the lecture.

Task 2: Appropriate Sampling from ERGMs

6 points

Let $\mathcal{G}$ the set of undirected, loopless graphs with $n = 3$ vertices and consider the exponential random graph model $(\mathcal{G}, P)$ containing only the *number of two-stars* statistic $g_1 = s_2$ with parameter $\theta_1 = \ln 2$.

According to the Gibbs sampling strategy defined in the lecture, specify the transition probabilities π in a Markov chain on $\mathcal{G}$ with unique stationary distribution P .

Note that you don't have to provide all 8×8 transition probabilities explicitly, since there is no need to distinguish between isomorphic graphs, and many transition probabilities are 0. Consequently, present your result within a single graph in which the nodes represent the equivalence classes of isomorphic networks in $\mathcal{G}$ and edges are labeled according to the positive transition probabilities.