University of Konstanz
Algorithmics Group
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Network Modeling
Winter Term 2011/2012

Assignments $\mathcal{N}^{\underline{o}}$ 3 - part i

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(solutions can be handed over at the beginning of the lecture)

Task 1: Hammersley Clifford Theorem — 10 points

Let $\mathcal{G}$ the set of undirected, loopless graphs with n vertices and let $c\colon V \to \{A, B\}$ divide the set of vertices $V = \{1, \ldots, N\}$ into two disjoint subsets, $V = A \uplus B$.

Consider the class of random graph models $\mathcal{K}_c = \{(\mathcal{G}, P)\}$ containing all models, which fulfill the following independence assumption.

> For all pairs of dyads d_1, d_2 it holds that d_1 and d_2 are conditionally independent, unless both of the following properties hold:
>
> - d_1 and d_2 are incident
> - all nodes incident to d_1 and d_2 belong to the same subset. More precisely, if $d_1 = \{u, v\}$ and $d_2 = \{x, y\}$, then
>
> $$c(u) = c(v) = c(x) = c(y) \ .$$

(a) Which random graph models in $\mathcal{K}_c$ are *Markov random graphs*?

(b) Provide a set of statistics, such that the resulting class of ERGMs is exactly the class $\mathcal{K}_c$. (cf. corollary)

(c) Let $V = \{1, 2, 3\} \uplus \{4\}$. Draw the *dependence graph* of $(\mathcal{G}, P) \in \mathcal{K}_c$.