

## Assignments $\mathcal{N}^o$ 1 - PART I

**released:** 31.10.2012     **due:** 07.11.2012, 13:30h  
(solutions can be handed over at the beginning of the lecture)

### Task 1: Structural Balance

**3 points**

Let  $\Delta$  the set of undirected triangle graphs, in which each edge is either labeled positive or labeled negative — that is, instead of being present or not, each of the three edges is either positive or negative. Define a random graph model on  $\Delta$ , such that all of the following conditions hold at the same time:

- (1) all balanced graphs are more probable than unbalanced ones
- (2) all edges are pairwise independent
- (3) every edge depends on the two others

### Task 2: Valid Probability Spaces

**2 points**

Use the binomial theorem to show that  $\mathcal{G}(n, p)$  is indeed a valid random graph model, i.e.  $\sum_{G \in \mathcal{G}} P(G) = 1$ .

### Task 3: Expected Number of Motifs

**4 points**

Again, consider the  $\mathcal{G}(n, p)$  random graph model:

- (a) What is the expected number of edges?
- (b) What is the expected number of triangles?
- (c) What is the expected number of induced  $k$ -circles?  
[induced  $k$ -circle:  $k$  nodes whose incident edges form a circle *without* shortcuts]