UNIVERSITY OF KONSTANZ
ALGORITHMICS GROUP
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Network Modeling
Winter Term 2012/2013

Assignments $\mathcal{N}^{\underline{\circ}}$ 2 - PART I

released: 14.11.2012 **due:** 21.11.2012, 13:30h
(solutions can be handed over at the beginning of the lecture)

Task 1: Defining an ERGM 4 points

An **e**xponential **r**andom **g**raph **m**odel is specified by a set of statistics and associated parameters. Together they define the probability of a graph as

$$P_\theta(G) = \frac{1}{\kappa(\theta)} \exp\left(\sum_{i=1}^{k} \theta_i \cdot g_i(G)\right)$$

where $g_i \colon \mathcal{G} \to \mathbb{R},\ \theta_i \in \mathbb{R},\ \theta = (\theta_1, \ldots, \theta_k)$ and the normalizing constant is implicitly defined by

$$\kappa(\theta) = \sum_{G' \in \mathcal{G}} \exp\left(\sum_{i=1}^{k} \theta_i \cdot g_i(G')\right)$$

Define an ERGM that is identical to the "boys&girls" planted partition model we used in the last assignments, and explain how the parameters of the two models translate into each other.

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Task 2: Dyad Dependency in ERGMs **6 points**

Let $\mathcal{G}$ the set of undirected, loopless graphs with $n = 3$ vertices and consider an exponential random graph model $(\mathcal{G}, P)$ with only one statistic, namely

(a) $t(G)$ (the number of triangles) with associated parameter value $\ln 2$.

(b) $m_a(G)$ (the number of edges connecting actors with the same attribute value) with associated parameter value $\ln 2$. In our case, let a divide the node set $\{1, 2, 3\}$ into *even* and *odd* numbers.

For each case separately, a) triangle statistic and b) homophily statistic, prove whether edge probabilities are dependent or independent.