University of Konstanz
Algorithmics Group
V. Amati / J. Lerner / D. Schoch

Network Modeling
Winter Term 2013/2014

Assignments $\mathcal{N}^{\underline{o}}$ 1

released: 30.10.2013 **due:** 04.11.2013, 9AM[1]

Task 1: Structural Balance 3 points

Let Δ the set of undirected triangle graphs, in which each edge is either labeled positive or labeled negative — that is, instead of being present or not, each of the three edges is either positive or negative. Define a random graph model on Δ, such that all of the following conditions hold at the same time:

(1) All balanced graphs are more probable than unbalanced ones.

(2) All edges are pairwise independent.

(3) Every edge depends on the two others.

Task 2: Characterizing properties of $\mathcal{G}(n, p)$ 3 points

Proof the following three properties of the $\mathcal{G}(n, p)$ model:

(1) The edge probability of every dyad is equal to p.

(2) The model is fully independent.

(3) There is just one model satisfying properties (1) and (2).

[1] If the tutorial is postponed to Wednesday, solutions can be handed in till Tuesday 12AM. Check the lecture homepage for changes.

Task 3: Expected Number of Motifs 3 points

Again, consider the $\mathcal{G}(n,p)$ random graph model:

(1) What is the expected number of edges?

(2) What is the expected number of triangles?

(3) What is the expected number of induced k-circles?
[induced k-circle: k nodes whose incident edges form a circle ***without*** shortcuts]

Task 4: R: Number of Triangles in $\mathcal{G}(n,p)$ 3 points

preparatory steps:

- Download the edgelist of the facebook graph from the lecture homepage and use it to create a network object in R.
- Implement the efficient algorithm to sample from a $\mathcal{G}(n,p)$
- Calculate p such that the expected number of edges is equal to the number of edges in the facebook graph (Hint: Task 3).

Use the calculated parameters of n and p to create 1000 network samples with your $\mathcal{G}(n,p)$ implementation. Count the number of samples, where more triangles are observed than in the facebook graph. What do you observe? Was this outcome expected?

Send your R-Script to `david.schoch@uni-konstanz.de`

A code example with additional hints can be found on the lecture homepage.