# Network Modeling

## Comparing TERGM with SAOM

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Winter 2014/2015
(last updated: January 7, 2015)

# Same input data but different modeling interest.

Snapshots $G_1, \ldots, G_T$ of a network evolving over time.

TERGM

Specifies $P(G_t | G_{t-1})$, i. e. the **conditional probability** of a network given the preceeding one.

SAOM

Specifies a **network-evolution process** that starts at $G_{t-1}$ and ends at a network with the same statistics as $G_t$.

# Can we mimick the purpose of the other model?.

TERGM

Markov-chain simulation can define a process starting at $G_{t-1}$ and "ending" at a network with the same statistics as $G_t$.

However, transition probabilities are not uniquely determined and stopping criterion is unclear.

SAOM

Process ends at $G_t$ with a certain probability
$\Rightarrow$ defines $P(G_t|G_{t-1})$.

However, probability depends on stopping criterion.

# Can a SAOM specify transition probabilities that are admissible for an ERGM Markov-chain?

Consider decision between $G^{(+e)}$ and $G^{(-e)}$, where $e = (u, v)$.

TERGM

Reversibility condition:

$$\frac{P(G^{(+e)})}{P(G^{(-e)})} = \frac{\pi^{(+e)}}{\pi^{(-e)}}$$

SAOM

Transition probabilities (up to rate function):

$$\pi^{(+e)} = \frac{P_u(G^{(+e)})}{\sum_{G' \text{ reach. from } G^{(-e)}} P_u(G')}$$

$$\pi^{(-e)} = \frac{P_u(G^{(-e)})}{\sum_{G' \text{ reach. from } G^{(+e)}} P_u(G')}$$

Define rate function $\lambda_u(G) = \sum_{G' \text{ reachable from } G} P_u(G')$.

# Can the ratio of the probability functions be the same?

## TERGM

$$P(G) \sim \exp\left(\sum_{i=1}^{k} \theta_i \cdot s_i(G)\right)$$

Statistics $s_i(G)$ evaluate the whole graph.

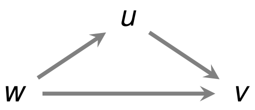
Transitive triplet for $G$:

## SAOM

$$P_u(G) \sim \exp\left(\sum_{i=1}^{k} \theta_i \cdot s_i(u; G)\right)$$

Statistics $s_i(u; G)$ evaluate the graph from the perspective of the active actor $u$.

No transitive triplet for $u$:



**Further differences.**

# Stationary distribution.

TERGM

Assumes that the Markov chain is in a stationary state (expected values of statistics do not change anymore).

Note that the Markov chain associated with an ERGM is a technical artefact which is not necessary to define the model.

SAOM

No assumption of stationarity (different stopping criterion).

# Synchronous tie-change events.

| TERGM | SAOM |
|---|---|
| Makes no assumptions in this respect. | Assumes that synchronous tie-change events cannot happen.<br><br>Assumes that individual tie-change events are conditionally independent, given the current state of the network. |

# Shorter-spaced observation intervals.

TERGM

Should benefit from more information (if the model is homogeneous over time).

SAOM

Have the constraint on the Jaccard coefficient
$\Rightarrow$ short intervals may be prohibitive if not enough tie-changes happen in between observations.