

# Network Modeling

## Comparing TERGM with SAOM

Viviana Amati    Jürgen Lerner

Dept. Computer & Information Science  
University of Konstanz

Winter 2015/2016  
(last updated: January 11, 2016)

# Same input data but different modeling interest.

Snapshots  $G_1, \dots, G_T$  of a network evolving over time.

## TERGM

Specifies  $P(G_t | G_{t-1})$ , i. e. the **conditional probability** of a network given the preceding one.

## SAOM

Specifies a **network-evolution process** that starts at  $G_{t-1}$  and ends at a network with the same statistics as  $G_t$ .

# Can we mimick the purpose of the other model?.

## TERGM

Markov-chain simulation can define a process starting at  $G_{t-1}$  and “ending” at a network with the same statistics as  $G_t$ .

However, transition probabilities are not uniquely determined and stopping criterion is unclear.

## SAOM

Process ends at  $G_t$  with a certain probability  
 $\Rightarrow$  defines  $P(G_t|G_{t-1})$ .

However, probability depends on stopping criterion.

# Can a SAOM specify transition probabilities that are admissible for an ERGM Markov-chain?

Consider decision between  $G^{(+e)}$  and  $G^{(-e)}$ , where  $e = (u, v)$ .

TERGM

SAOM

Reversibility condition:

$$\frac{P(G^{(+e)})}{P(G^{(-e)})} = \frac{\pi^{(+e)}}{\pi^{(-e)}}$$

Transition probabilities  
(up to rate function):

$$\pi^{(+e)} = \frac{P_u(G^{(+e)})}{\sum_{G' \text{ reach. from } G^{(-e)}} P_u(G')}$$

$$\pi^{(-e)} = \frac{P_u(G^{(-e)})}{\sum_{G' \text{ reach. from } G^{(+e)}} P_u(G')}$$

Define rate function  $\lambda_u(G) = \sum_{G' \text{ reachable from } G} P_u(G')$ .

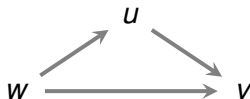
# Can the ratio of the probability functions be the same?

TERGM

$$P(G) \sim \exp \left( \sum_{i=1}^k \theta_i \cdot s_i(G) \right)$$

Statistics  $s_i(G)$  evaluate the whole graph.

Transitive triplet for  $G$ :



SAOM

$$P_u(G) \sim \exp \left( \sum_{i=1}^k \theta_i \cdot s_i(u; G) \right)$$

Statistics  $s_i(u; G)$  evaluate the graph from the perspective of the active actor  $u$ .

No transitive triplet for  $u$ :

**Further differences.**

# Stationary distribution.

## TERGM

Assumes that the Markov chain is in a stationary state (expected values of statistics do not change anymore).

Note that the Markov chain associated with an ERGM is a technical artefact which is not necessary to define the model.

## SAOM

No assumption of stationarity (different stopping criterion).

# Synchronous tie-change events.

## TERGM

Makes no assumptions in this respect.

## SAOM

Assumes that synchronous tie-change events cannot happen.

Assumes that individual tie-change events are conditionally independent, given the current state of the network.



## Shorter-spaced observation intervals.

### TERGM

Should benefit from more information (if the model is homogeneous over time).

### SAOM

Have the constraint on the Jaccard coefficient  
⇒ short intervals may be prohibitive if not enough tie-changes happen in between observations.