Assignments $\mathcal{N}^{\underline{o}}$ 2

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For this exercise sheet, assume a partition of the node set V into 'girls' and 'boys': $V = V_1 \uplus V_2$, and consider the set $\mathcal{G}$ of all *directed*, loopless graphs with vertex set V.

Task 1: Homophily Model   10 points

Let $p \in (0,1)$ be a real number between zero and one and let p_h be a real number such that $0 < p + p_h < 1$. We consider the random graph model on $\mathcal{G}$ with the properties

(a) the model is fully independent;

(b) the edge-probability of every mixed-gender dyad (boy→girl or girl→boy) is equal to p;

(c) the edge-probability of every same-gender dyad (girl→girl or boy→boy) is equal to $p + p_h$.

For this model

(1) Write down the formula for the probability of a graph.

(2) Assume you have an observed network $G_{\text{obs}} \in \mathcal{G}$. Calculate the maximum likelihood estimates for the two parameters (p and p_h).

(3) Explain: what does it intuitively mean if the parameter p_h is less than, equal to, or greater than zero?

(4) What are the expected number of mixed-gender edges and the expected number of same-gender edges?

Task 2: R: Number of triangles in $\mathcal{G}(n, p)$ **10 points**

Preparatory steps:

- Download the edgelist of the facebook graph from the lecture home-page and use it to create a network object in R.

- Implement the efficient algorithm to sample from a $\mathcal{G}(n, p)$

- Calculate p such that the expected number of edges is equal to the number of edges in the facebook graph.

Use the calculated parameters of n and p to create 1000 network samples with your $\mathcal{G}(n, p)$ implementation. Count the number of random samples, where more triangles are present than in the observed facebook graph. What do you observe? Was this outcome expected?