

Assignments \mathcal{N}^o 6

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Task 1: Exponential random variable

3 points

Let T an exponential random variable with probability density function

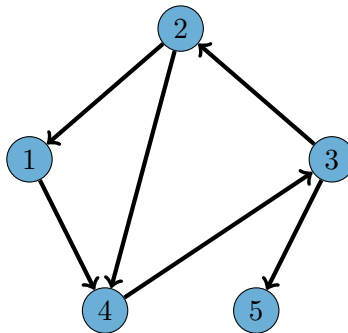
$$\varphi_T(t) = \lambda e^{-\lambda t}, \lambda > 0, t > 0$$

where λ is the rate parameter. Prove the memoryless property of T .

Task 2: Chain probability

7 points

Consider the following network with 5 nodes



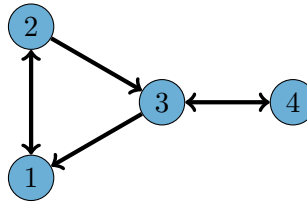
Let us assume that Actor 4 has the opportunity to make a change. His decision is based on an objective function including the following statistics: *outdegree*, *reciprocity*, *transitive triplets* and *three-cycle*. The values of the corresponding parameters are $\beta_{out} = -1.5$, $\beta_{rec} = 2.5$, $\beta_{tran} = 0.8$ and $\beta_{cyc} = -0.1$.

Compute the chain probability for Actor 4.

Task 3: Chain probability in R**10 points**

Write the following two functions in R:

- (a) The function *netstats* should return the outdegree and the number of reciprocal dyads for an actor i . The arguments of the functions are an adjacency matrix x and an actor id i . The output must be a two dimensional vector returning the values of the outgoing and reciprocal ties statistics.
- (b) The function *evalfct* should return the vector of probabilities of all possible changes that an actor i can make. The arguments of the function are an actor id i , an adjacency matrix x and a vector β of the statistical parameters for outdegree and reciprocal dyads.
- (c) Create the adjacency matrix of the following network

Set $\beta_{out} = -1$ and $\beta_{rec} = 1.2$.

- i. Perform a micro-step for actor 3, i.e. calculate the tie change probabilities and flip the tie to actor j^* with the highest probability.
- ii. Calculate the tie change probabilities for j^* to all other actors.