University of Konstanz
Department of Computer & Information Science
Sabine Cornelsen

Design and Analysis of Algorithms
Winter 2018/2019

Assignment 1

Post Date: Oct 22 2018 **Due Date:** Oct 29 2018, 11:30 am
You are permitted and encouraged to work in groups of two.

Problem 1: Growth of Functions — 8 Points

Rank the functions

$$n \log n,\ \binom{2n}{4},\ 2^n,\ 4n^4,\ n^n,\ 16^{\log_2 n},\ \log(n!), n!$$

by increasing order of growth, i.e., find an order $f_1, \ldots, f_7$ with $f_1 \in \mathcal{O}(f_2), \ldots, f_6 \in \mathcal{O}(f_7)$, and prove the correctness of your ranking.

Problem 2: Recurrence Equations — 5 Points

Give a Θ-bound for the following recurrences:

(a) $T_1(n) = 2 \cdot T_1\left(\frac{n}{2}\right) + \sqrt{n}, \quad T_1(1) = 1$

(b) $T_2(n) = T_2(n-1) + 2(n-1), \quad T_2(1) = 1$

Problem 3: Divide and Conquer — 7 Points

Let n points in the plane be given. You may assume for simplicity that no two points have the same x-coordinate. Develop a divide-and-conquer algorithm that finds a pair of points with the smallest Euclidean distance between them. Analyze the run time of your algorithm. Can you achieve a run time in $\mathcal{O}(n \log n)$?

Hint: It may be helpful to think first about the 1D version of the problem. How can the merge step be realized in linear time in 2D?