University of Konstanz
Department of Computer & Information Science
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Design and Analysis of Algorithms
Winter 2018/2019

Assignment 2

Post Date: 29 Oct 2018 **Due Date:** 05 Nov 2018, 11:30 am
You are permitted and encouraged to work in groups of two.

Problem 1: Fraud Detection — 6 Points

Assume you have n bank cards. Each bank card is associated with a bank account. Different bank cards might be associated with the same bank account. The associated bank is stored encrypted on your bank card. However, you have a device that can check whether two bank cards are associated with the same bank account. The task is to find out whether there is a set of at least $n/2$ bank cards associated with the same bank account. Show how to decide this question by comparing at most $\mathcal{O}(n \log n)$ pairs of bank cards?

Problem 2: Threshold — 4 Points

Give the greatest $a \in \mathbb{N}$ such that Algorithm A with running time

$$T_A(n) = a \cdot T_A\left(\frac{n}{9}\right) + n^2$$

is asymptotically faster than Algorithm B with running time

$$T_B(n) = 11 \cdot T_B\left(\frac{n}{3}\right) + n^2.$$

Problem 3: Regularity Condition — 5 Points

Let $a \geq 1$, $b > 1$, and $f : \mathbb{N} \to \mathbb{R}_{>0}$. Show that $f(n) \in \Omega(n^{\log_b a + \epsilon})$ for some $\epsilon > 0$ if there is a $c < 1$ and an $n_0 \in \mathbb{N}$ such that $a \cdot f(\lceil n/b \rceil) \leq c \cdot f(n)$ for all $n \geq n_0$.

Problem 4: Selection — 5 Points

Does the algorithm Select still work in linear time if the input elements are divided into groups of 3 and 7, respectively?