UNIVERSITY OF KONSTANZ
DEPARTMENT OF COMPUTER & INFORMATION SCIENCE
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Assignment 4

Post Date: 12 Nov 2018 **Due Date:** 19 Nov 2018, 11:30 am
You are permitted and encouraged to work in groups of two.

Problem 1: Union-Find with Path Compression — 5 Points

Consider FIND with the following alternative path compression: After traversing the path from a vertex to its root, we update the parent pointer of each vertex along the path to point to its grandparent. Consider, e.g., subpath

$$i \to j \to k \to l \to \cdots$$

Performing FIND(i) with alternative path compression results in k being predecessor of i and l being predecessor of j. Direct successors of the root keep the root as predecessor.

Go through the proof of the *Theorem of Hopcroft & Ullman* and find the inferences that require FIND to be implemented with path compression. Is the proof still correct if the alternative path compression is used?

Problem 2: Independent Vertex Sets — 5 Points

Let $G = (V, E)$ be a graph. Let $\mathcal{I} = \{V' \subseteq V;\ \{u, v\} \notin E \text{ for } u, v \in V'\}$.

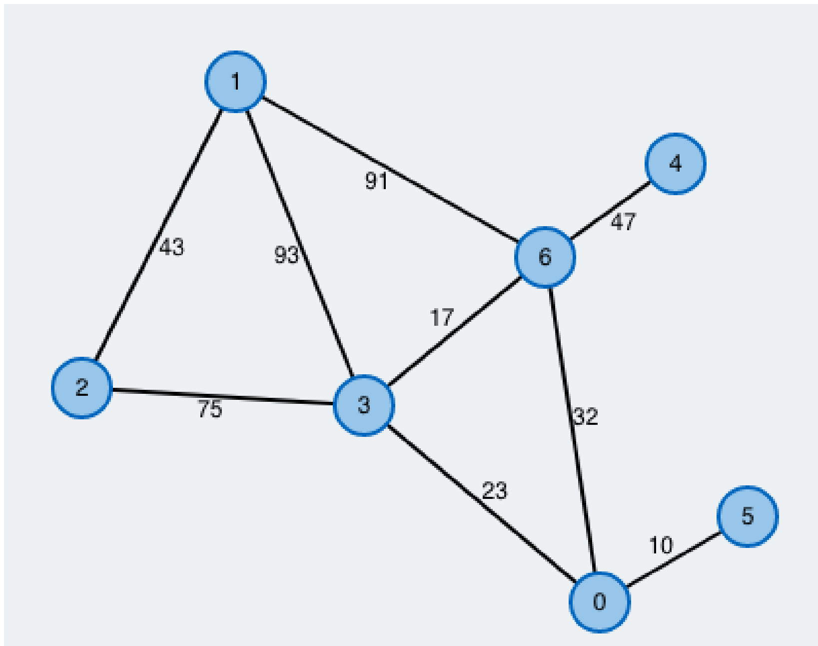
(a) Show that $(V, \mathcal{I})$ is an independence system.

(b) Is $(V, \mathcal{I})$ also a matroid? Justify your answer.

[please turn over]

Problem 3: Minimum Spanning Tree **4 Points**

Create manually the minimum spanning tree of the graph below. Explain your algorithm and why it is an application of red and blue rules.



Problem 4: Unique Minimum Spanning Trees **6 Points**

(a) Prove: Let G be a connected graph with real-valued edge weights. If for each cut the crossing edge with the lightest weight is unique, then G has a unique minimum spanning tree.

(b) Does the inverse of the implication in (a) hold?

(c) Prove or disprove: If in a connected graph G with real-valued edge weights all edges have pairwise distinct edge weights, then G has a unique minimum spanning tree.

(d) Does the inverse of the implication in (c) hold?