University of Konstanz
Department of Computer & Information Science
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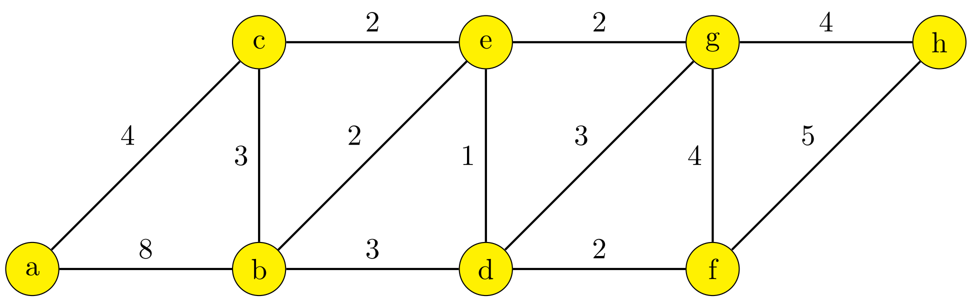
Design and Analysis of Algorithms
Winter 2018/2019

Assignment 6

Post Date: 26 Nov 2018 **Due Date:** 03 Dec 2018, 11:30 am
You are permitted and encouraged to work in groups of two.

Problem 1: Minimum Cut — 6 Points

(a) Find a minimum cut for the following graph with the algorithm of Stoer and Wagner. Start with vertex a and comment each step of the algorithm.



(b) Which conclusion in the proof of correctness of the algorithm of Stoer and Wagner requires non-negative edge weights?

(c) Find an example graph with negative edge weights such that the algorithm of Stoer and Wagner does not find a minimum cut.

Problem 2: All-Pairs-Min-Cut — 6 Points

Let $G = (V, E)$ be a graph with n vertices and edge weights $c : E \to \mathbb{R}$. For two vertices s and t in G let λ_{st} be the weight of a minimum s-t-cut in G.

(a) Let $v_1, v_2, \ldots, v_k$ be a sequence of vertices in G. Show that

$$\lambda_{v_1,v_k} \geq \min\{\lambda_{v_1,v_2}, \lambda_{v_2,v_3}, \ldots, \lambda_{v_{k-1},v_k}\}.$$

(b) Consider now the complete graph with vertex set V and edge weight $-\lambda_{u,v}$ on the edge $\{u, v\}$, $u, v \in V$. Let T be the minimum spanning tree on the thus weighted complete graph. Show that for any pair of vertices $s, t \in V$ there is an edge $\{u, v\}$ in T with $\lambda_{s,t} = \lambda_{u,v}$.

(c) Conclude that the $\frac{n(n-1)}{2}$ pairs of vertices $s, t \in V$ have at most $n-1$ distinct values λ_{st}.

[please turn over]

Problem 3: Crossing Cuts **4 Points**

Let $G = (V, E)$ be a graph with edge weights $c : E \to \mathbb{R}_0^+$ and let λ be the weight of a minimum cut of G. Let $(S, V \setminus S)$ and $(T, V \setminus T)$ be two minimum cuts of G such that none of the four *corners* $S \cap T$, $S \setminus T$, $T \setminus S$ and $V \setminus (S \cup T) =: \overline{S \cup T}$ is empty. Proof that $c(S \cap T, \overline{S \cup T}) = c(S \setminus T, T \setminus S) = 0$ and $c(S \cap T, S \setminus T) = c(S \setminus T, \overline{S \cup T}) = c(\overline{S \cup T}, T \setminus S) = c(T \setminus S, S \cap T) = \lambda/2$.

Problem 4: Vertex Capacities **4 Points**

Let $((V, E); s, t; c)$ be an extended flow network where not only edge capacities, but also vertex capacities are constrained, i. e., $c : E \cup V \to \mathbb{R}_0^+$ and a flow $f : E \to \mathbb{R}_0^+$ must satisfy, in addition to the edge capacity constraint and the flow conservation constraint, the following vertex capacity constraint

$$\sum_{w:(w,v) \in E} f(w, v) \leq c(v) \text{ for all } v \in V.$$

Show that determining a maximum flow in a network with additional vertex capacities can be reduced to determining a maximum flow in a network with only edge capacities.