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Department of Computer & Information Science
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Design and Analysis of Algorithms
Winter 2018/2019

Assignment 11

Post Date: 14 Jan 2019 **Due Date:** 21 Jan 2019
You are permitted and encouraged to work in groups of two.

Problem 1: FPTAS — 5 Points

Consider an optimization problem Π with the following properties.

(a) An instance of Π is a vector with integer entries.

(b) The value of any feasible solution is a non-negative integer.

(c) The optimum value is bounded by a polynomial in the size of the input and the maximum absolute value of any integer occuring in the input, i.e., there is a polynomial p with $v_{\text{OPT}}(x) < p(|x|, x_{\max})$ for any instance x of Π, where $v_{\text{OPT}}(x)$ is the value of an optimum solution, $|x|$ is the size of x and $x_{\max}$ is the maximum absolute value of the entries in x.

Prove that there is a pseudo-polynomial time algorithm for Π if there is an FPTAS for Π.

Problem 2: Vertex Cover Relaxation — 5 Points

Consider the LP relaxation of Vertex Cover:

$$\begin{array}{rll} \text{minimize} & \sum_{v \in V} x_v & \\ \text{subject to} & x_u + x_v \geq 1 & \{u, v\} \in E \\ & x_v \geq 0 & v \in V \end{array}$$

Prove: There exists an optimal solution to the LP relaxation of Vertex Cover such that $x_v \in \{0, 0.5, 1\}$ for all vertices $v \in V$.

[Please turn over]

Problem 3: Euclidean Traveling Salesman Problem **10 Points**

Consider the Euclidean Traveling Salesman problem (Euclidean TSP) parameterized by the number of points in the interior of the convex hull of a given set of points:

Euclidean Traveling Salesman:

Input: set $V \subset \mathbb{R}^2$ of n points, threshold $\ell \in \mathbb{R}_{\geq 0}$

Parameter: number k of points of V in the interior of the convex hull of V

Question: Is there a round trip of length at most ℓ?

Recall that a *round trip* is an ordering $v_1, \ldots, v_n$ of the points in V. The *edges* of a round trip $v_1, \ldots, v_n$ are the line segments $\overline{v_1 v_2}, \ldots, \overline{v_{n-1} v_n}, \overline{v_n v_1}$. The length of a round trip is the sum of the lengths of the edges on the round trip. Two edges *cross* if their intersection is finite and non-empty and does not consist only of common endpoints of the edges. For simplicity, you may assume in the following that no three points in V lie on a single line.

(a) Show that no two edges of a shortest round trip cross.

(b) Show that the points on the boundary of the convex hull appear consecutively in a shortest round trip.

Let $p_1, \ldots, p_{n-k}$ be the points on the boundary of the convex hull in cyclic order. Let V_I be the set of points in V that are in the interior of the convex hull of V. We fix an arbitrary ordering $q_1, \ldots, q_k$ of the points in V_I. We are looking for the length of the shortest round trip that contains $p_1, \ldots, p_{n-k}$ and $q_1, \ldots, q_k$ in this order as subsequences.

Let $F_p(i, j)$ and $F_q(i, j)$, respectively, be the length of a shortest p_1-p_i- and p_1-q_j-path that consist of the points $p_1, \ldots, p_i$ and $q_1, \ldots, q_j$ and respect their orderings.

(c) Find recursive formulas for $F_p(i, j)$ and $F_q(i, j)$, and show that $F_p(i, j)$ and $F_q(i, j)$ can be computed in $\mathcal{O}(nk)$ time using dynamic programming.

(d) Express the length of the shortest round trip that has $p_1, \ldots, p_{n-k}$ and $q_1, \ldots, q_k$ as subsequences in terms of $F_p(i, j)$ and $F_q(i, j)$.

(e) Conclude that Euclidean TSP is fixed-parameter tractable for parameter k. What running time do you achieve when computing the shortest round trip with this approach?