University of Konstanz
Department of Computer & Information Science
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Design and Analysis of Algorithms
Winter 2018/2019

Assignment 12

Post Date: 21 Jan 2019 **Due Date:** 28 Jan 2019, 11:30
You are permitted and encouraged to work in groups of two.

Problem 1: MultiCut in Trees — 4 Points

The problem *MultiCut in Trees* takes as input a tree $T = (V, E)$, a set $H \subseteq \binom{V}{2}$ of pairs of vertices, and a threshold k. It asks whether it is possible to separate the pairs in H by removing at most k edges from T, i.e., whether there is a set $E' \subseteq E$ of size at most k such that s and t are in different connected components of $T - E := (V, E \setminus E')$ for any pair $(s, t) \in H$.

Show that *MultiCut in Trees* with the threshold k as parameter is fixed-parameter tractable.

Problem 3: Covering Points with Lines — 6 Points

The problem Line Cover is defined as follows:

Input: A set of points $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ in the plane.

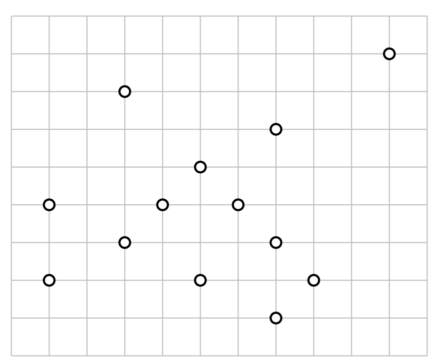
Parameter: $k \in \mathbb{Z}_{\geq 0}$

Output: Is there a set L of at most k lines such that each point $p_i \in P$ lies on a line L?

(a) Consider the instance of Line Cover given on the right. Show that the points can be covered with 3 lines.

(b) Prove: Any line containing at least $k + 1$ points of P must be included in the solution of Line Cover with parameter k.

(c) Show that Line Cover parameterized by k is fixed-parameter tractable.



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Problem 2: Solving the LP-Relaxation of Vertex Cover **10 Points**

Show how to use a max-flow algorithm in order to find an optimum solution for the LP-relaxation of vertex cover. To this end consider the following steps for a graph $G = (V, E)$.

Let $G' = (A \cup B, E')$ be the bipartite graph that is constructed from G as follows. For each vertex $v \in V$ there are two vertices $a_v \in A, b_v \in B$, and for each edge $\{u, w\} \in E$, there are the edges $\{a_u, b_w\}, \{a_w, b_u\} \in E'$.

(a) Let V' be a minimum vertex cover of G'. Show that

$$x_v := \begin{cases} 1 & \text{if } \{a_v, b_v\} \subseteq V' \\ 0 & \text{if } \{a_v, b_v\} \cap V' = \emptyset \\ 0.5 & \text{else} \end{cases}$$

is an optimal solution to G's LP relaxation of Vertex Cover.

Consider the following flow network D: Direct the edges of G' from A to B. Add a source s with edges to all $a \in A$ and and a sink t with edges from all $b \in B$ to G'. All capacities are one. Let f be a maximum integer s-t-flow on D. Let $M = \{e \in E(A, B);\ f(e) > 0\}$.

(b) Show that $|V'| \geq |M|$ for any vertex cover V' of G'.

Let $S = \{v \in V;\ \text{there is a directed } s\text{-}v\text{-path in } D_f\}$. Let $V' = (A \setminus S) \cup (B \cap S)$.

(c) Show that V' is a vertex cover of G'.

(d) Show that each vertex in V' is incident to an edge in M.

(e) Show that no edge in M is incident to two vertices in V'.